Combination of GPS Carrier Phase data with a Calibrated Time Transfer Link

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Abstract**—*We discuss the necessity and the method of the combination that keeps the advantages of TW and GPS carrier phase. TAI time transfer network is highly redundant. This method gives a new approach to use the redundancy at the baseline level. The method and the related conclusion are suitable for the combination of TW and GPS PPP.*

I. Introduction

A. General

The GPS carrier phase measurements (CP hereafter) are two orders of magnitude more precise than the GPS code data, much less sensitive to multi-path and allow to better estimate the atmosphere effects. The CP only solution can not be used in time transfer because of the one-to-one correlation between the phase ambiguity and the clock value as well as its discontinuity in case of tracking interruption. The GPS code data may be used as an absolute scale. Several methods have been proposed to combine the code and CP data; all these solutions are of the direct processing of GPS P code and GPS CP data. This is called the *direct-combination* in this paper.

The time transfer techniques presently used to generate the TAI are TWTT (Two Way Time Transfer, TW hereafter), GPS two frequency multi-channel P3 code (P3 hereafter) and GPS single-frequency multi or single channel C/A codes (C/A hereafter). Mathematically the GPS CP information can be combined with any of these calibrated TAI time links. Here the ambiguity, discontinuity in the CP is cancelled or greatly reduced meanwhile the advantages of different techniques are kept. Taking TW as an example, it is calibrated and the measurement may be performed with sub-ns accuracy and long term stability. However TW measurements are disturbed by the diurnal variation of 1-2.5 ns peak to peak (Fig. 1) and its resolution is poor (a point over 2 hours). On the other hand, the CP has no calibration but very high resolution (30s) and better frequency stability. The main idea of the proposed approach is to use the derivatives supplied by CP to improve the TW link function. Combining the independent observable CP with an absolutely calibrated and determined time link is called *post-combination* in the paper because the combined compositions are independently measured and calculated before the combination. There exist many techniques enabling to combine a series of observation and its derivatives. The Vondrak-Cepek combined smoothing, its mathematical principles, smoothing coefficients and transfer functions are discussed in this paper.

A series of careful designed numerical tests were made using simulated and measured data. We prove that the *post-combination* of CP with C/A or with P3 codes data can greatly improve the short term stability w.r.t. the code-only solutions and has an accuracy similar to that of the direct-combinations such as the IGS, AIUB and PPP etc; the combination of CP and TW considerably reduces the diurnal disturbance in TW KU band link (hereafter TW KU) and improves the short term stability. The drift in CP only solution gives no visible influence to the link results. With a careful configuration of the weights, the smoothing coefficients and the filters, it is possible to combine the CP with more than two types of absolute links, such as TW and P3. This allows us to improve both the uB and uA uncertainties in the TAI time transfer. An application of combining TW and GPS is to reduce the densification of the TW measurement which is expensive and laboured compared to GPS.

BIPM now routinely calculates PPP [8]. The dominating short term stability in PPP is assigned by CP. The derivatives used for the combination can also be derived from PPP. The method discussed here is suitable for combining TW and PPP.

B. A strategy to improve the TAI

The TAI international time transfer network is composed of 56 national laboratories. Each possesses atomic clocks and one or several time transfer equipments. A third of them, contributing the most part of TAI clocks, possess the geodetic receivers which produce both the P3 code and CP data.

At present the time transfer observables used to generate the TAI are TW (19%), P3 (12%) and C/A (67%) as well as internal link (2%). However the TAI international time transfer network is highly redundant [1]. Roughly speaking, only one third of the total observables are used for the Circular T computation. Among the redundant observables, the CP data are available at 30% of the laboratories that possess the geodesic receivers. However, the CP data are never used for TAI. It should be pointed out that these 30% laboratories contribute about 80% of TAI atomic clocks and all the

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Primary Frequency Standards (PFS) including all the Cs fountains. This implies that if we improve the time transfer quality for these 30% labs, the gain in TAI is 80%. An effective strategy to improve the TAI is to use the high accurate CP data. The CCTF 2006 encouraged the use of CP in time and frequency transfer.

The CP data are largely used in geodetic positioning. This allows the accuracy of the relative positioning to reach the millimetre level or some 10 ps in time unit. In 1990, Schildknecht et al. [3] started to study the use of CP data in time transfer. Several authors have contributed to this study and software have been developed for this purpose [2], [4], [5], [6], [7]. In these solutions, the accuracy potential of the CP is fully used. The time stability up to 10-day averaging is expected to be about 0.1-0.2 ns level. One of the solutions is PPP. Technically, there are no major difficulties to use PPP in TAI time transfer [8].

However, things are not so simple. Among the 30% labs possessing CP, half of them contribute about two third of TAI clocks and the PFS perform the TW as their primary time transfer technique. TW, due to its symmetric observation procedure that allows a direct cancel of the atmosphere effects (dominant error source in time transfer) and its certainty of long term stability, has its technical advantages. GPS is for them a back up.

A natural question is why not combining TW and PPP? One of the reasons disfavours this kind of combination is the probably existing biases and long term instability in the GPS P codes. It is true the P codes piloting the tendency of the PPP would be less stable than TW as an absolute reference. A combination of PPP and TW may improve the short term stability but meanwhile might reduce the long term stability.

As we can easily understand, the higher short term stability in PPP comes from the CP and the long term instability comes from the P codes, their variations and calibration. This leads us to think why not combining TW and CP? This combination should take the advantages of one side the short term stability and high resolution of CP and the other side the free-atmosphere observations with long term stability of TW. This is the goal of this paper.

Recent studies prove this TW+CP combination is not only possible but also necessary: (a) CP solution is computable [6], [7]; (b) [2] proves the existence of the diurnal disturbance in the TW KU links (major TW observation in TAI). Fig.1 is a typical example. Strong diurnal signals of 1-2.5 ns peak to peak are visible in the KU baseline. The TDev on the bottom left supports the hypothesis of the existence of the 1-day periodic term. Further investigation (section III) shows they do not come from the clocks and the same signal does not exist in CP. This implies the possibility to remove or at least to reduce the diurnal disturbance by combining the two observations. Because TW and GPS are two completely independent techniques, their combination makes the result less noisy and meanwhile less biased and more robust.

In the numerical test, we discuss also the combinations of CP with P3 code and CP with C/A code. The combination of CP and P3 gives accuracy similar to that of the PPP's. The meaning of these tests is to show how flexible the method to combine different observations and how can the CP doubly reduces the instability of the codes. Comparisons between the direct-combination (PPP) and post-combinations can also evaluate the methods used.

The Vondrak-Cepek combined smoothing [9] is used for the post-combination. We had taken a quick review of the method and presented some first results in [2] and here a detailed discussion.

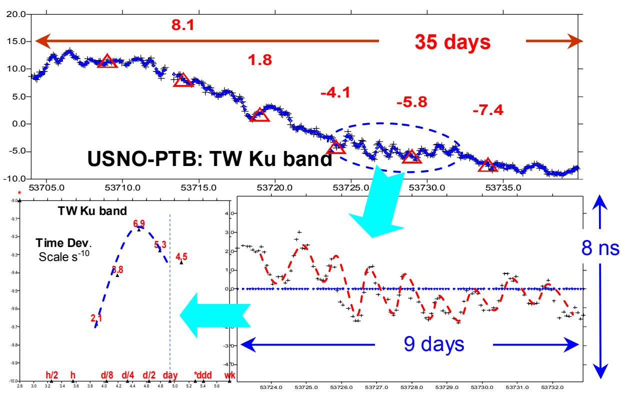


Fig. 1 USNO-PTB TW KU baseline, up to 2 ns peak to peak diurnal signal is visible. The left plot on the bottom is the Time Deviation of the measured points in the zoomed plot on the bottom right. Strong 1-day tendency is clear

II. MATHEMATICAL PRINCIPLES

Let us consider a series of TW time transfer observations $y(t)$ as a function of time, the derivative $y'(t_i)$ at a time epoch t_i can be defined by $[y(t_{i+1})-y(t_i)]/(t_{i+1}-t_i)$. Suppose we have another independent series of CP observations $Y(t)$, its derivative is $Y'(t) = [Y(t_{i+1})-Y(t_i)]/(t_{i+1}-t_i)$. The physical meaning of the two sets of derivatives is the same: the rate of the difference of the two end clocks: $y'(t_i) = Y'(t_i)$. Therefore,

$$\int_{Mjd1}^{Mjd2} y'(TW)dt = \int_{Mjd1}^{Mjd2} Y'(CP)dt \qquad (1)$$
$$TW_{\mathrm{MJD\,i+1}} = TW_{\mathrm{MJD\,i}} + \Delta(TW_{\mathrm{MJD\,i}}) = TW_{\mathrm{MJD\,i}} + \sum_{j=\mathrm{MJD\,i}}^{\mathrm{MJD\,i+1}} \Delta(CP_j)$$

The interval of the CP observation is usually much smaller than that of TW: 30 to 300 seconds. Equation (1) implies that a TW observation at epoch MJD_{i+1} can be represented by its previous observation value taken at MJD_i plus the sum of a series of the small increases of CP during the period. The discrepancy is the total observation error of the integrating period. By minimizing the discrepancy, we can establish the constrain condition of the combination to determine the smoothed curve. The first author designed a mathematical model to make the combination that is under testing.

In fact, there are existing methods to smooth a function with given derivatives. One of the successful methods is that of Vondrak-Cepek. In equation (1), if we put the $\Delta(CP_j)$ over its observing interval, we have the derivative of CP on MJD_j. Our problem becomes a typical Vondrak-Cepek combined smoothing [9]. This method is based on the Whittaker-Robinson-Vondrak smoothing that removes the high frequency noises present in a series of unequally spaced observations. The idea is to find the compromise between the smoothness of the searched curve on the one hand and the 'fidelity' of this curve to the observed values on the other hand. This, namely '*original-smoothing*', has been used in the TAI production. In 2000, the original-smoothing was improved to take into account both the observation and its derivative values that are assumed to be independent from each other and to be de-correlated between the individual observations in a same series.

The new method consists in finding a weighted compromise among three different conditions: 1) condition ***S***: smoothness of the searched curve (analytically unknown) as in the original-smoothing; 2) condition ***F***: its fidelity to the observed function values and 3) condition ***F***°: its fidelity to the observed first time derivatives. A least square adjustment is then made by minimizing these three constraints, i.e. the expression:

$$Q = S + \varepsilon F + \varepsilon^\circ F^\circ = \min$$

$$\varepsilon = \frac{(2\pi f)^6 T}{1-T};\ \varepsilon^\circ = \frac{(2\pi f)^4 T^\circ}{1-T^\circ} \qquad (2)$$

Here, $\varepsilon \geq 0$ is the smoothing coefficient of the function ***F*** and $\varepsilon^\circ \geq 0$ is the smoothing coefficient of the derivative ***F***°. The dimensions are respectively time^{-6} and time^{-4}. Each is expressed as its function of filter: ***T*** ($0 \leq T \leq 1$) and ***T***° ($0 \leq T^\circ \leq 1$) are respectively the ratios of the amplitude of smoothed curve over that of observed curve for a periodic function with the period: $P=1/f$. f is the frequency.

$$T \ or \ T^\circ = \frac{Amp_{\ smooth}}{Amp_{\ observed}} \qquad (3)$$

Table 1 Relation between transfer function (***T*** and ***T***°), smoothing coefficient (ε/day^{-6} *and* ε°/day^{-4}) and period length (***P***/day)

T /***T***°		**0.10**	**0.30**	**0.50**	**0.80**	**0.99**
P=.5	***ε***	438E+03	169E+04	394E+04	158E+05	390E+06
	ε°	277E+01	107E+02	249E+02	997E+02	247E+04
P=1	***ε***	684E+01	264E+02	615E+02	246E+03	609E+04
	ε°	173.	668.	156E+01	623E+01	154E+03
P=5	***ε***	0.438	1.69	3.94	15.8	390.
	ε°	0.277	1.07	2.49	9.97	247.
P=10	***ε***	684E-05	264E-04	615E-04	0.246	6.09
	ε°	173E-04	668E-04	156	0.623	15.4
P=90	***ε***	129E-10	496E-10	116E-09	463E-09	115E-07
	ε°	264E-08	102E-07	238E-07	950E-07	235E-05

Table 1 gives the relation between the smoothing coefficients (ε *and* ε°), the transfer functions (***T*** and ***T***°) and the period length (***P***). ε *and* ε° increase separately with ***T*** and ***T***°, and decrease with ***P***. From equation (2), when ε *and* ε° decrease to zero, the combination becomes the 'original-smoothing', with only the observation values used while the derivative values are in consequence ignored. If we know the period of the observed function, we can fix the value ***P*** to look for the optimal combination by adjusting the values of ***T***, ***T***° and ε, ε° based on the related uncertainties. Our goal is to combine the CP and TW in which we know the disturbing signal is diurnal. We set ***P***=1 and the ratio of the transfer functions of ***T***/***T***° to be about 1/3, i.e. ***T***=0.3 and ***T***°=0.99, according roughly to the ratio of the short-term uncertainties of CP over TW. From Table 1, we have ε =264000 and ε° =154000. Fig. 2 shows the variation of $\varepsilon/\varepsilon^\circ$ as function of ***T***/***T***° for **P**=1. Another adjustable parameter in the combination is the weight. The method allows weighing the two observables (the function and its derivative) on every epoch, based on the related uncertainties. Usually the uncertainty for a type of TAI link is constant and we use in the following tests equal weight. It is interesting to point out that the original purpose of the Vondrak-Cepek combination is rather to find the periodic signal in a series of observations by removing the noises. In our application, we want not only to reduce or limit the measurement noises but also the periodic signal that exists in one series but not in the other. Careful setting of $\varepsilon/\varepsilon^\circ$ and ***T***/***T***° is necessary. Fortunately, the period ***P***=1 is known in our case.

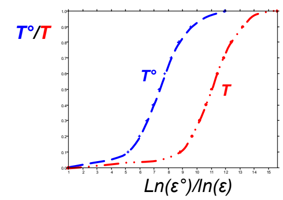


Fig. 2 Transfer functions of filter (***T*** and ***T***°) as functions of the smoothing coefficients (ε *and* ε°) with fixed period ***P***=1 day

III. NUMERICAL TESTS

We designed a series of tests based on the statistic knowledge of time transfer practices. The method is simple: first we do a combination and compare the result to the true or a well known value. We then calculate the *gain*=(σ_{obs}-σ_{comb})/σ_{comb}. Here σ_{obs} is the standard deviation of differences between observation and the known value; σ_{comb}. is that between the combined and the known values. Obviously, the error in the known values is included in the gains. We are therefore not too optimistic. The known values come from 1) the simulation with the result known beforehand; 2) USNO-PTB, i.e. the highest redundant baseline in TAI network. In addition to the usual data of TW KU, P3, C/A, PPP and CP, we have also the TW X band (TW X hereafter) link. Its uncertainty uA is about 0.2 ns better than that of KU and less affected by the diurnal disturbance [10], at least no found over the test data set: 623 points between Mjd 53702-53730. Time Deviation (TDev) is used to further figure the periodic signal. Our main goal is to combine TW KU and CP.

A. Test 1: Simulated data

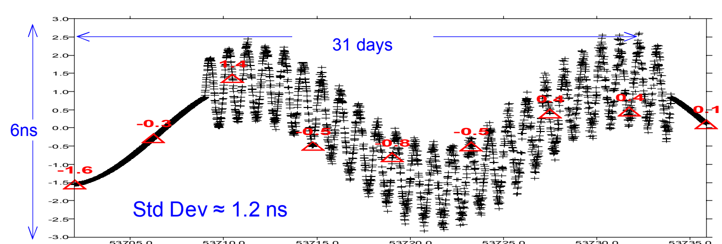


Fig. 3A Simulated link with the added white noises and diurnal disturbances

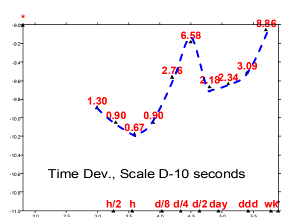


Fig. 3B TDev corresponding Fig. 3A

The characters of the simulated link:

- Clock differences = Sin(2πf)+Cos(2πf)+n *(cf. Fig. 4A,4C)*
- TW noise ~ Normal distribution: $N(0,\sigma)$, σ =0.5 *ns*
- CP noise ~ Normal distribution: $N(0,\sigma)$, σ =0.1*ns*
- *Diurnal variation amplitude: 1 ns (peak to peak 2 ns)*

The total observation error is about 1.2 ns. Fig. 4A, 4C plot the simulated link without errors. Fig. 3A and 3B show the link with the mixed errors and its TDev where the diurnal disturbance are clearly present (only 25 days over totally 31 days between MDJ 53709-53734, to simulate the fact that the diurnal signals are sometimes strong and sometimes weak or even absent).

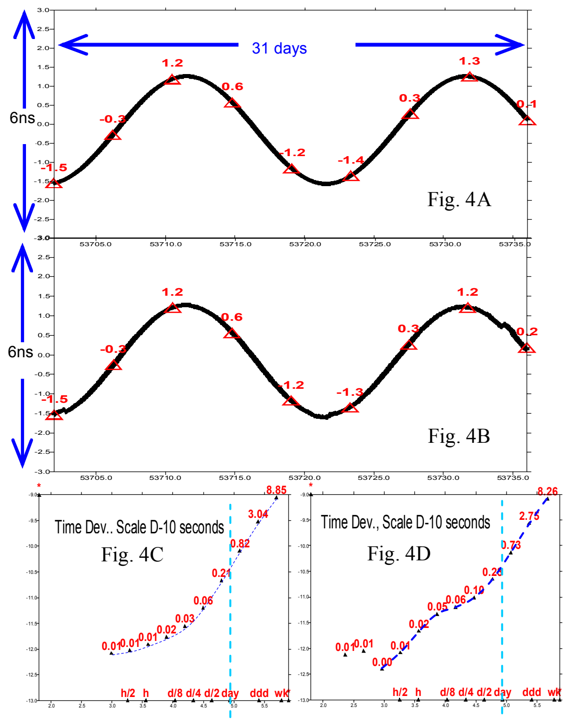


Fig. 4 A is the simulated time link; B is the CP+TW combined link; C is the TDev of A; D is the TDev of B

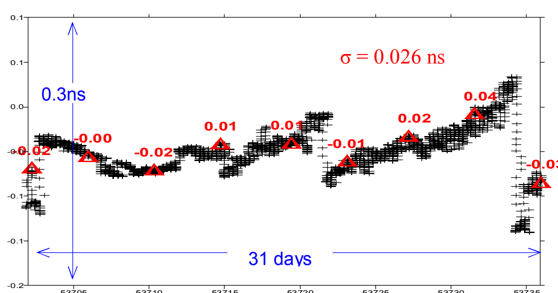


Fig. 5 Combination error: the differences between Fig. 4A and 4B with the standard deviation σ = 0.026 ns

Fig. 4A and 4C are the simulated time link (analytically known) and its TDev. Fig. 4B and 4D are the Vondrak-Cepek combined link result and its TDev. They are very close to each other. Comparing Fig. 4A to 4B, we see that the total errors of 1.2 ns, composed of the normal distributing errors plus the periodic terms, are almost completely removed. Fig. 5 shows the differences between the known value (Fig. 4A) and the combined value (Fig. 4B). The biggest residuals appear on the two limits of the data set. The standard deviation is 0.026 ns that can be considered as the total combining error. This proves the parameter setting and the combining computation have been well done.

B. Test 2: Combination of GPS CP and TW KU

Fig. 1 shows the presence of the diurnal disturbance in TW KU. Fig. 6 shows the TDev of TW X and CP links for the same period. The two links agree with each other with no diurnal signal observed. We can combine KU and CP to reduce the diurnal signal and compare the result to TW X link.

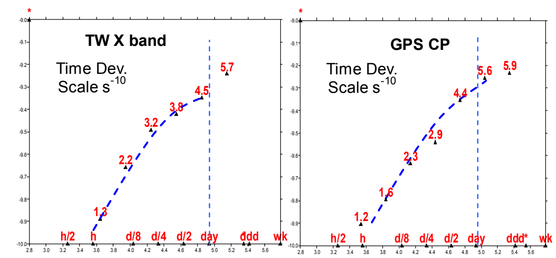


Fig. 6 TDev of TW X band and TDev of GPS CP data over the same period of MJD 53723-53732 as shown in Fig. 1. No diurnal signals present

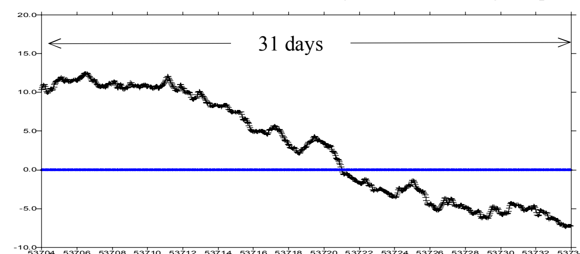
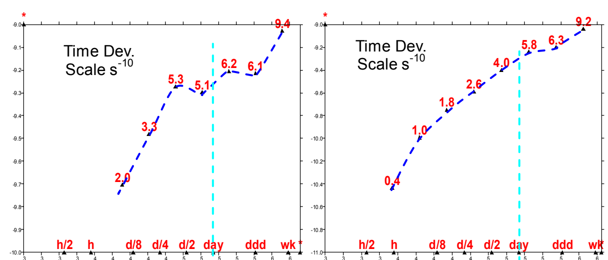


Fig. 7 Combination of CP and TW KU links w.r.t. Fig. 1 (USNO-PTB: between MJD 53704-53734)



Fig; 8 TDev before and after the combination with CP. The left is the TDev of the KU link in Fig. 1; the right is that of the combined link in Fig. 7

Fig. 7 displays the CP and KU (the same link and period as shown in Fig. 1) combined link. The diurnal signals originally in KU have disappeared. This is supported by the TDev in Fig. 8. The strong diurnal signal in KU link (left) disappeared (right) after the combination with CP. The standard deviations of the differences between KU and X are respectively σ=**0.716**ns before and σ=**0.291**ns after the combination with CP. The gain is 146%. We can foresee further amelioration will be obtained to combine CP and TW X. USNO-PTB is the most "heavy" time link, through which about one fourth of the total TAI clocks transferred. Any amelioration in this link will be benefited directly by TAI.

C. Test 3: Combination of GPS CP+C/A and CP+P3

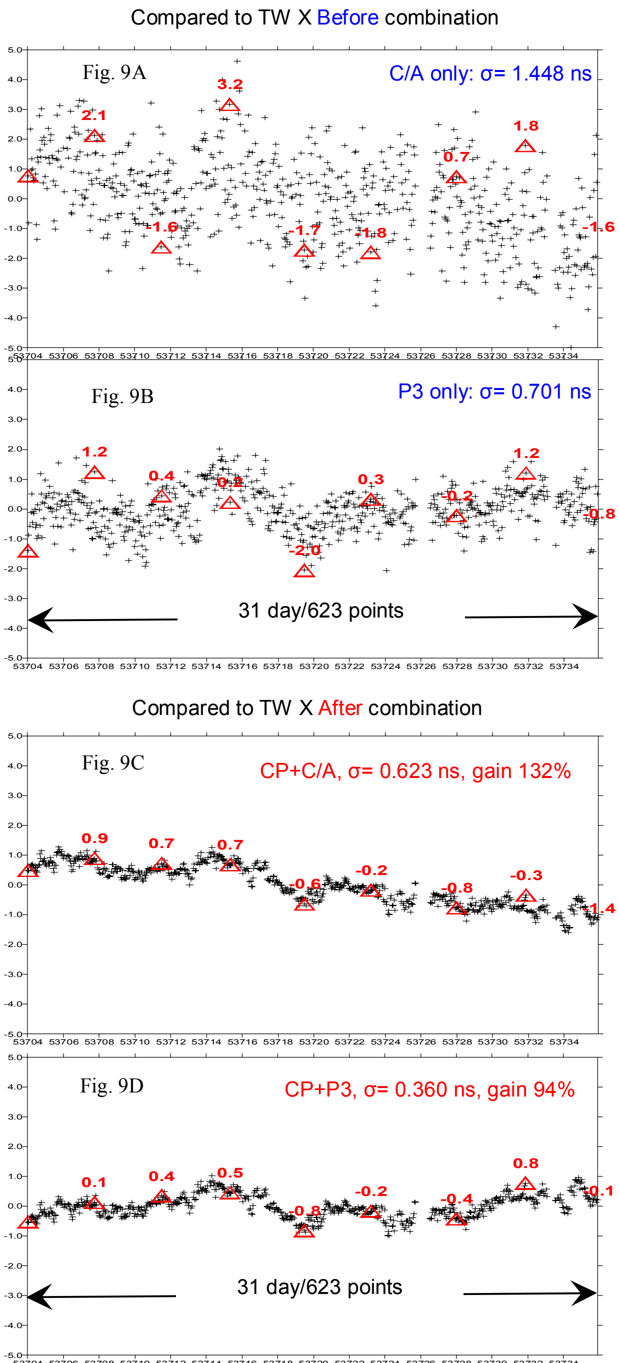


Fig. 9 C/A and P3 links compared to TW X before and after to be combined with CP (USNO-PTB: 623 points between MJD 53704-53734)

Combining CP and P3 code should output similar result as PPP. Combining CP and C/A code keeps the calibration information in the C/A code and greatly reduces the short-term instability caused by the measurement noises, the multi-path and the atmosphere delays in the codes. Comparing the results to that of TW X or the direct-combination (PPP here cf. section D), we examine the methods. If the combination agrees to TW X and to PPP within a certain tolerance, we may conclude that both the method and the computation are correct.

Fig. 9 compares the C/A and P3 code links to TW X link before and after their combinations with CP. The comparisons were made using always the USNO-PTB baseline over the 623 points between MJD 53704-53734. For the C/A code (Fig. 9A and 9C), the standard deviations of the differences between C/A code and X are respectively σ=**1.448**ns before and σ=**0.623**ns after combining with CP. The gain is 132%.

For the P3 code links (Fig. 9B and 9D), the standard deviations of the differences with respect to TW X are σ=**0.701**ns before and σ=**0.360**ns after the combination with CP. A gain of 94% is obtained. For the same data set, the standard deviations of the differences between PPP and X is σ= 0.354ns. This shows the *direct*-combination and *post*-combination give similar uncertainties. See also section D.

D. Test 4: Difference of the direct and post combinations

PPP is the solution of *direct*-combination of CP with the P1 and P2. The observation interval is 5 minutes. P3 is the ionosphere delay free linear combination of P1 and P2. P3 and C/A data are in the CGGTTS format with the interval of about 16 minutes. We combined CP with C/A or P3 separately and compared the combination result to the PPP solution. Fig. 10A and 10B show that the code only solutions (scattered blue points) are very noisy compared to the PPP solution (black in the middle). The standard deviation of the differences of C/A and PPP is σ= 1.273 ns; that of P3 is σ= 0.701 ns. After the combination with CP (Fig. 10C and 10D), the deviations are reduced respectively to σ= 0.381 ns and σ= 0.230 ns, within the PPP uncertainty. The gains are respectively 132% and 204%. The differences between PPP and CP+P3 come partially from the data set used: PPP is based on the receiver readings of the locking epochs while P3 is based on the every 13-minute-smoothed value over the 16-minute-interval (3 minute readings are then skipped every 16 minute).

E. Test 5: Long-term stability of the combination

The CP solution is affected by a drift [2]. Because we use the derivative of the CP, the drift, if it is not very important, should not affect the combined result. The reason is that the derivative series is piloted by the calibrated link. Table 2 is 4 months' comparisons of the TW X to the combinations of CP with different absolute links. About 40% gains are obtained for different observations of KU, C/A and P3.

Table 2 Long term comparison /ns (MJD 53645-53765)

Link	**σ** (Link-TW X)	**After** combining CP σ (Link-TW X)	**Gain**
TW **KU** band	0.90	0.64	40%
GPS **C/A** code	1.66	1.15	44%
GPS **P3** code	0.98	0.71	38%

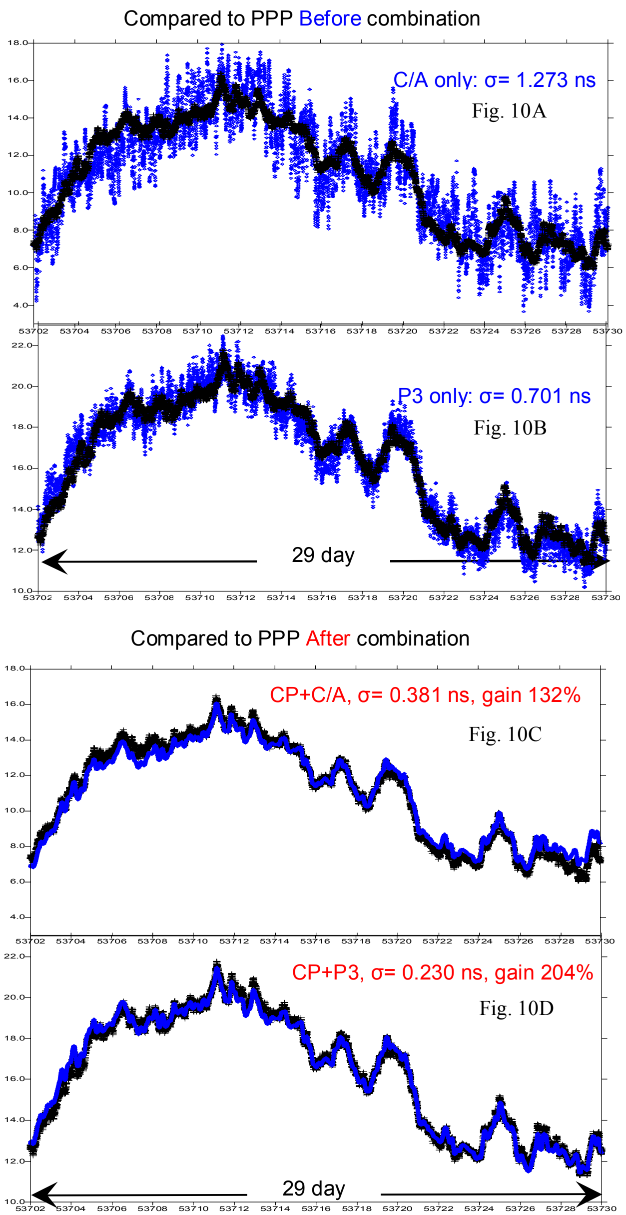


Fig. 10 Comparisons between the *direct*-combination (PPP: black curve) and the *Post*-combination (C/A and P3: blue point/curve). Fig; 10A, 10B are before the combination with CP and 10C, 10D are after the combination. (USNO-PTB: 623 points between MJD 53704-53734)

IV. CONCLUSION

GPS CP information can be combined with any calibrated time link, namely the Post-combination, in which, the absolute calibration is kept; the short term stability is greatly improved; the *diurnal disturbance* in TW KU is suppressed. We studied the *Vondrak-Cepek* method and used it for the combination.

GPS and TW are completely independent techniques. Their combination makes the solution *less noisy, less biased and more robust*. Simulating test shows 95% errors of both the white noises and the diurnal disturbances are removed. Measuring data analysis shows that the *differences to TW X band are doubly reduced* after the combinations.

The *Post-combination* of CP+P3 agrees with PPP within 0.25 ns, i.e. smaller than the PPP uncertainty.

The TW labs, contributing 2/3 of the total TAI clocks, have the GPS CP available. *Use of CP combining to TW is an effective strategy to improve TAI.*

BIPM routinely produces PPP since this year [8]. The dominant contribution to the short term stability in PPP is assigned by CP. The derivatives of the clock difference used in the combination can also be derived by PPP. All the discussion concerning CP is suitable for PPP which, unlike CP, is continued without the accumulated drift. Further analysis is required.

The TW observation interval is 2 hours while that of GPS is 30s. For short and middle terms (< 3 days), GPS PC or PPP is more advantageous than TW. An application of the combination of TW + GPS enables reducing the densification of the TW measurements which is much more expensive and laboured w.r.t. GPS. Easy to know that this method allows bridging a lost TW link calibration due to ground or satellite operations.

Finally, the TAI worldwide network is *highly redundant*. We propose a method to combine the advantages of the two fundamental link techniques: TW and GPS, on baseline level.

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